SECTION A (56 Marks)

Answer ALL questions in this section. ALL working for each question must be shown clearly.

1. (a) Simplify

(i) $(A \cup B)' \cap A$

(ii) $(A - B) \cup A$ (4 marks)

(b) In a class of twenty pupils, there are twelve who study English but not History, four who study History but not English and one who study neither English nor History. How many pupils study History? (3 marks)

2. (a) Given the three points A (4, 0), B(0, 2) and C(-2, -2), show that AB = BC. ($2\frac{1}{2}$ marks)

(b) A square ABCD is formed where A, B and C are points given in 2. (a) above and D is the fourth point whose coordinates are not known. Calculate the coordinates of D. ($4\frac{1}{2}$ marks)

3. (a) (i) Prove that $(1 - \cos A)(1 + \sec A) = \sin A \tan A$.

(ii) Eliminate θ if,

$$x = 4 \sec \theta$$

$$y = 4 \tan \theta$$ (4 marks)

(b) Given that $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, show that the composition function is not commutative. (3 marks)

4. (a) Given that $y = \frac{\sin x - \cos x}{\sin x + \cos x}$,

find $\frac{dy}{dx}$ (4 marks)

(b) Differentiate $4x^2y + x + xy = 0$. (3 marks)

5. Find:

(a) $\int \cos^3 x \, dx$ (4 marks)

(b) $\int x^2 (1 - x)^2 \, dx$ (3 marks)

6. (a) The displacement vector $\underline{a}$ maps A(3, -2) on to B(-1, 3) and $\underline{b}$ maps A on to C(4, 5). Find $\underline{a} + \underline{b}$.

(4 marks)

(b) Show that the vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ is at right angles to the straight line

$$\mathbf{r} = (\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) + s(2\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}).$$ (3 marks)

7. (a) (i) Find the value of $\dfrac{15!}{12!\,3!}$

(ii) How many arrangements can be made of the letters in the word TROTTING? ($2^1/_2$ marks)

(b) (i) Write down the statement which is true among the following:

- If $P^2 = 0$, then $P = 0$
- If $PQ = 0$, then $P = 0$ or $Q = 0$.
- If $P = 0$ or $Q = 0$, then $PQ = 0$.

(ii) Solve the following by the matrix methods:

$2x = 5 + y$

$3 + 2y + 3x = 0$

($4^1/_2$ marks)

8. (a) In a geometrical progression, the sum of the second and third terms is 6, and the sum of the third and fourth terms is −12. Find the first term and the common ratio. (4 marks)

(b) Obtain the first four terms in the expansion of $\left(1 - \dfrac{x}{2}\right)^6$ (3 marks)

SECTION B (44 Marks)

Answer any FOUR (4) questions from this section. ALL workings for each question answered must be shown clearly.

9. In the following data, the lengths of 40 leaves are recorded to the nearest millimeters:

138	164	150	132	144	125	149	157
146	158	140	147	136	148	152	144
168	126	138	176	163	119	154	165
146	173	142	147	135	153	140	135
161	145	135	142	150	156	145	128

(a) Construct a frequency distribution with class intervals 118 – 126, 127 – 135, 136 – 144 etc. (3 marks)

(b) Determine the mean length using the formula

$$\bar{X} = A + \frac{\Sigma fd}{N},$$

where A = assumed mean
d = the deviation of any class mark from the assumed mean.
N = total frequency. (4 marks)

(c) Calculate the median for the lengths of the leaves. (4 marks)

10. (a) Two events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$.

Evaluate:

(i) $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P(A' \cap B')$.

Another event C is such that A and C are independent, and
$P(A \cap C) = \frac{1}{12}$, $P(B \cup C) = \frac{1}{2}$.

Show that B and C are mutually exclusive. ($8\frac{1}{2}$ marks)

(b) Two balls are taken out at random from a bag containing 6 red and 2 blue balls. Find the probability that just one ball is blue. ($2\frac{1}{2}$ marks)

11. (a) Verify that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology. (6 marks)

(b) Let p be " Juma reads Daily News."

Let q be "Juma reads Mzalendo"

let r be "Juma reads Nipashe"

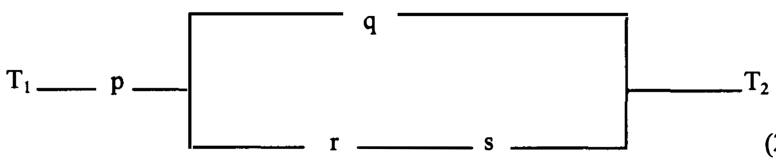
Give a verbal sentence which describes each of the following:

(i) $(p \vee q) \wedge \sim r$

(ii) $(p \wedge q) \vee \sim (p \wedge r)$

(iii) $\sim (p \wedge \sim r)$ (3 marks)

(c) Construct a compound statement that corresponds to this network.



(2 marks)

12. (a) Find the smallest root of the equation $x^2 - 4x + 2 = 0$ by using the algorithm $x_{n+1} = \frac{1}{4}(x_n^2 + 2)$ starting with $x_o = 0.5$. Calculate x_1 and x_2 only. (9 marks)

(b) If you were told to continue the algorithm in (a) above when would you stop the iterations? (2 marks)

13. (a) If $z = 3 - i$ express $z + \frac{1}{z}$ in the form $a + bi$, where a and b are real. (3 marks)

(b) If $z = x + yi$ and $\bar{z}$ is the conjugate of z, find the values of x and y such that $\frac{1}{z} + \frac{2}{\bar{z}} = 1 + i$. (4 marks)

(c) Given that $2 + 3i$ is a root of the equation $z^3 - 6z^2 + 21z - 26 = 0$, find the other roots. (4 marks)

14. (a) The radius r of a circle is 5 cm. Find the increase in the area A of the circle when the radius expands by 0.01 cm. (4 marks)

(b) A particle P moves in a straight line such that its distance S m from a fixed point O at time t s is given by $S(t) = 9t^2 - 2t^3$. What is the velocity and acceleration of a particle P when $t = 3$ s?

Find also the distance of P from O when $t = 4$ s and show that it is then moving towards O. (7 marks)

$(2+3i)(2-3i)$
$4+9=13$

$-3-9$

125

216

21
4

$z^2+7z+10$
$(z^2+2z+5z+10)$
$z(z+2)\,5(z+2)$

13
4